

Index Numbers

(Relevant to AAT Examination Paper 4: Business Economics and Financial Mathematics)

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In business world, we need to reduce a large number of facts or information to data so that we can identify changes over time. An index number provides a standardized way for comparing the levels of prices, wages, volume of output and other variables at given dates to their levels at a previous date. An index number can also be used to compare one thing to another. In conclusion, index numbers make it easier to interpret data. This article shows you how to present and interpret data in index number format.

Example 1

Suppose the retail sales for RGE shop in 2009 are \$256 billion and the sales in 2010 are \$257 billion. The increase of \$1 billion is significant. But if we convert the sales data to an index which is based on sales in 2009, as shown below, the increase is not that significant as it is less than 4%.

$$\begin{aligned}\text{Retail sales in 2010 / Retail sales in 2009} &= \$257 \text{ billion} / \$256 \text{ billion} \times 100 \\ &= 100.39\end{aligned}$$

Example 2

The GDPs of China and Hong Kong are US\$4,814 billion and US\$2,088 billion respectively. What is the GDP of China compared with that of Hong Kong?

The index of GDP for China is 2.305 compared with that of Hong Kong, and is calculated as

$$(\text{GDP of China} / \text{GDP of Hong Kong}) = (4,814 \text{ billion} / 2,088 \text{ billion}) = 2.305$$

In other words, the GDP of China is 2.305 times that of Hong Kong.

Prices indices and quantity indices

An index can be a price index or a quantity index. A price index measures the change in price of a group of items over time. A quantity index measures the change in quantity of a group of items over time. For example, the food price index is 168 in 2010 compared with 150 in 2009. The index increased by 18 points between 2009 and 2010; this is a rise of $(18/150) \times 100 = 12$ percent. These figures are often defined as price relatives since they indicate how the price changes over time.

From the above discussion, the index number is expressed in term of percentages but the percent sign is usually omitted. The base period should be stated. In the above example for food prices, we used 2009 as the base period.

Weighted index

In the business, we may be required to deal with a basket of commodities. Weighted means of relative indices are always used. To obtain a weighted mean of relative indices, we need to calculate index relatives for each component and then to obtain a weighted average of the relatives using the corresponding weights.

Example 3

The wholesale price index in PP shop is made up of the prices of three items. The price index and weighting in 2009 and 2010 for each item are as follows.

Table 1 Price and weight information for PP shop

Item	Price, 2009	Price, 2010	Weight
Bread	\$2.00	\$4.00	60
Tea	\$10.00	\$12.00	20
Cookies	\$2.00	\$2.50	40

Calculate an index for prices of PP shop for year 2010.

Solution

Price index:

$$\text{Bread} \quad 4/2 \quad = 2.00$$

$$\text{Tea} \quad 12/10 \quad = 1.20$$

$$\text{Cookies} \quad 2.5/2.0 \quad = 1.25$$

$$\begin{aligned} \text{PP shop's price index} &= (2.00 \times 60 + 1.20 \times 20 + 1.25 \times 40) / (60 + 20 + 40) \\ &= 194/120 \\ &= 1.62 \end{aligned}$$

Choice of weight

There are two methods in computing a weighted index: the Laspeyres method and the Passche method.

Laspeyres index

Laspeyres indices use base period values as weights. The original prices and quantities of the items are used in calculating the index in terms of either price or quantity consumed.

Passche index

Passche indices use current year values as weight. The current prices and quantities of the items are used in calculating the index in either terms of price or quantity consumed.

Example 4

DOC Company produces and sells four types of electric appliances. The prices and quantities in 2009 and 2010 are shown below:

Computation of Laspeyres and Passche index for DOC Company, using 2009 as the base period:

Table 2 Price and quantity information for DOC Company

	2009		2010	
Type	Price (\$)	Quantity ('000)	Price (\$)	Quantity ('000)
Radio	100	20	120	15
Toaster	200	40	250	25
Clock	130	30	130	50
Hair dryer	225	10	250	10

Calculate the Laspeyres price index and Passche quantity index to measure the overall changes between 2009 and 2010.

Solution:

Laspeyres price index

$$\frac{\sum p_t q_0}{\sum p_0 q_0} \times 100 = \frac{120 \times 20 + 250 \times 40 + 130 \times 30 + 250 \times 10}{100 \times 20 + 200 \times 40 + 130 \times 30 + 225 \times 10} \times 100 = 116.41$$

First we use the quantity in 2009 as our base in calculating Laspeyres price index for the total amount spent on the four items in DOC Company. We conclude that the price for DOC Company has increased by 16.41 percent.

Passche quantity index

$$\frac{\sum p_t q_t}{\sum p_t q_0} \times 100 = \frac{120 \times 15 + 250 \times 25 + 130 \times 50 + 250 \times 10}{120 \times 20 + 250 \times 40 + 130 \times 30 + 250 \times 10} \times 100 = 90.69$$

The Passche quantity index uses current quantity as weights instead of base year price as weights (as the Laspeyres index does). The result indicates that there has been a decrease of 9.31 percent in the quantity of electrical appliances between 2009 and 2010.

When you are deciding which type of index to use, the following points should be considered.

Table 3: Differences between the Laspeyres and Passche indices

Laspeyres index	Passche index
The index requires weights for the base year only.	The index requires weights by time to time and it is costly
The index can be computed when either the current prices or quantities are known	The index cannot be computed until the current prices and quantities are known.
The index value can be directly compared over several years.	The index value can only be compared between the current year and the base year.
The index does not reflect current changes over time because the weights are out-of-date.	The index reflects current information.

It should be noted that the Laspreyes index is more commonly used in today's business. The weights should be revised every few years.

Time series deflation

Another type of price index used for measuring the real value of a commodity is the consumer price index, or CPI. This index is widely used in both the private and public sectors. In the public sector, governments use the CPI to reflect changes in the cost of living for residents. In the private sector, employers often use the CPI in calculating their employees' salary rises. The

CPI is considered the most relevant inflation measure from the point of view of consumers. For example, say the cost of a commodity was \$100 in 2009 and \$120 in 2010; this represents an increase of 20 percent. If the retail price index is 205 percent between 2009 and 2010, we can argue that the real cost of the commodity has increased.

Consider the data shown in Table 4. Paul works for City Ltd. Over the last three years, he has earned a monthly salary increase of \$500 each year. Though his manager claims that \$500 is equivalent to the increase in the cost of living over the past three years, Paul has found that his salary rise has not kept pace with inflation. The real value of Paul's salary is only \$18,000 in year 3 compared with \$20,000 in year 1.

Table 4 : The real salary

Year	Monthly salary	Consumer Prices Index (CPI)	Calculation	Real salary
1	\$20,000	150		\$20,000
2	\$20,500	160	$20,600 / 160 \times 150$	\$19,219
3	\$21,000	175	$21,000 / 175 \times 150$	\$18,000

When comparing the costs and profits of company or assessing investment decisions, we need to consider the "real money" values. The best way to analyze the situation is to calculate the real money values before making our decisions.

Another example, given below, illustrates how to use interest rates to calculate inflation in order to assess investment or future profits.

Example 5

The PPW Company is considering a three-year project with the following expected profits and inflation index. The initial investment is \$300,000 and the required rate of return is 20 percent. Determine whether the project should be accepted.

Table 5: Inflation index and expected profit

Year	Inflation index	Expected profit (\$'000)
0	100	(300)
1	95	100
2	102	130
3	111	125

Solution

The inflation index provides an index (column 2) and this can be used to deflate the expected profits (column 3) and convert them to their real values. For example, the real profit in year 1 is \$90,000 only instead of \$99,000.

Year (1)	Inflation index (2)	Expected profit (\$'000) (3)	Calculation (4)	Real profits (\$'000) (5)
0	100	(300)		(300)
1	110	99	$(99/110) \times 100$	90
2	115	138	$(138/115) \times 100$	120
3	120	144	$(144/120) \times 100$	120

The net present value is the difference between the present value of the cash outflows and the present value of future cash inflows. The total profit is given in column 4 and compared with the initial investment. The net present value of the three-year project, as shown in the table, is negative, so PPW Company should not invest in this project.

Net present value

Year (1)	Real profits (\$'000) (2)	Discount factor at 20% (3)	Present value (4)
0	(300)	1.000	(300)
1	90	0.833	74.97
2	120	0.694	83.28
3	120	0.579	69.48
		NPV	(76.27)