Concepts in Investments – Risks and Returns
(Relevant to PBE Paper II – Management Accounting and Finance)

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In PBE Paper II, students are expected to demonstrate a comprehensive knowledge of the risk and return relationship for individual securities and a portfolio of securities. However, the December 2012 examiner's report included the comment that students were not familiar with the basics of investment. This article highlights the fundamental concepts of stock investments, in particular, the risk and return relationship, as well as with its arithmetic representation.

Returns

Investors invest their money into businesses with a hope of getting returns. In a stock investment returns come in two forms: (i) **capital gains**, which refers to an appreciation of the investment value; and (ii) **dividend yield**, which shows the amount of dividend paid out by an entity each year relative to its share price. If Investor A purchases a share in Entity A for $100 on 1 January 2013 and the share price rises to $105 at 31 December 2013, the percentage capital gain is 5% (($105-$100)/$100). If Entity A distributes a $2 dividend per share during the year ended 31 December 2013, the dividend yield is 2% ($2/$100). The total return on the investment is then 7%, which is the sum of the two percentages.

The annual return on a particular entity's stock fluctuates because the stock price depends on a number of external factors (e.g. the macro-economic environment and government policy) and internal factors (e.g. introduction of new products to the market). It is therefore advisable to average the returns in order to obtain a fair picture of the typical return expected by investors. There are basically two approaches to averaging returns: (i) arithmetic average (or mean); and (ii) geometric average (or mean).

Calculating the **arithmetic mean** is simple and straightforward. An investor need only (i) add the returns in the past years; and (ii) divide the total by the number of years. The **geometric mean**, on the other hand, assumes that there is a **fixed annual return** which would compound over the period to the terminal value. The terminal value, assuming there are four years' returns denoted by r_1 r_2 r_3 and r_4, can be found by the following formula:

$$Terminal\ value = (1 + r_1)(1 + r_2)(1 + r_3)(1 + r_4)$$

Applying the assumption of geometric mean,

$$(1 + geometric\ mean)^4 = Terminal\ value = (1 + r_1)(1 + r_2)(1 + r_3)(1 + r_4)$$

Rearranging the above equation,

$$\boxed{\boldsymbol{Geometric\ mean = \sqrt[4]{(1 + r_1)(1 + r_2)(1 + r_3)(1 + r_4)} - 1}}$$

Example:
Investor A is planning to invest in Stock A which had returns of 5%, -10%, 12%, 7% and 4% in the past 5 years. Calculate the average return of Stock A by arithmetic mean and geometric mean.

$$Arithmetic\ mean = \frac{5\% + (-10\%) + 12\% + 7\% + 4\%}{5} = 3.6\%$$

$$Geometric\ mean = \sqrt[5]{(1+5\%)(1-10\%)(1+12\%)(1+7\%)(1+4\%)} - 1 = 3.33\%$$

Risks

Stock investments present a number of risks to investors. There is no promise that an investor can get their money back. There is no guarantee that the directors of the entity declare dividends in a year. Further, as indicated in the above example, there is a downside risk for stock prices (in one of the years, the return on Stock A was -10%). There is therefore considerable uncertainty in stock investments, and this depends on the **dispersion** or **spread** of the possible returns. The more variable the returns, the greater the risk will be. To measure dispersion (therefore risk), one has to calculate the variance (or standard deviation) of the return.

Variance measures the spread of possible returns and is represented by the average of squared deviations around the mean:

$$\boldsymbol{Variance = \sum_{i=1}^{S} p_i (R_i - Mean)^2}$$

This formula represents the variance of the entire population. From the statistical point of view, if the number of years (i.e. samples) is small, the variance is obtained by dividing the sum of squared deviation around the mean by (n – 1) instead of n, assuming n is the number of years (i.e. samples). In general, the larger the variance, the greater the fluctuations in returns will be and therefore the greater the risk in investing in a specific instrument.

Another measure of risk is known as the **standard deviation** which represents the **square root of variance**.

$$\boldsymbol{Standard\ deviation = \sqrt{Variance}}$$

Example:
Following the above example, calculate the variance and standard deviation of the returns of Stock A. Assume the arithmetic mean is used.

	Return	Mean	Deviation around the mean	Squared deviation
Sample 1	5%	3.6%	1.4%	0.00020
Sample 2	-10%	3.6%	-13.6%	0.01850
Sample 3	12%	3.6%	8.4%	0.00706
Sample 4	7%	3.6%	3.4%	0.00116
Sample 5	4%	3.6%	0.4%	0.00002
			Summation of squared deviation	**0.02692**

Variance $_{\text{Stock A}}$ = 0.02692 / (5 -1) = 0.00673

Standard deviation $_{\text{Stock A}}$ = $\sqrt{0.00673}$ = 8.2%

Portfolio of securities

A portfolio of securities may include a number of stocks. One of the reasons for constructing a portfolio is that it leads to the benefit of diversification which reduces variability (i.e. risk). Before exploring this benefit, it is important to first calculate the portfolio return, which is relatively straightforward. Portfolio return is the weighted average of the returns of individual security:

$$\boldsymbol{Portfolio\ return = w_1R_1 + w_2R_2}$$

As diversification through a portfolio reduces risk, portfolio variance is not necessarily the same as the weighted average of the variances of individual security. Indeed, calculating portfolio variance involves the concept of **covariance** and **correlation coefficient**. Diversification works because different stocks behave differently and do not move exactly together. If two stocks move together, they are said to be positively correlated. Conversely, if two stocks move in exactly the opposite direction (i.e. when one stock performs well the other performs badly), they are said to be negatively correlated. Covariance is the measure of how two variables change together:

$$\boldsymbol{Covariance_{12} = \sum_{i=1}^{S} p_i(R_{1i} - Mean_1)(R_{2i} - Mean_2)}$$

Example:

Investor A finally determined to invest in a portfolio which consists of 30% in Stock A and 70% in Stock B. Stock B had returns of 1%, 13%, 3%, -8% and 1% in the past 5 years. Calculate the weighted arithmetic average of the portfolio and the covariance of the returns of Stocks A and B.

$$Arithmetic\ mean\ of\ return\ of\ Stock\ B = \frac{1\% + 13\% + 3\% + (-8\%) + 1\%}{5} = 2\%$$

$$Portfolio\ returns = (30\%)(3.6\%) + (70\%)(2\%) = 2.48\%$$

	Stock A			Stock B			Deviation of Stock A x Deviation of Stock B
	Return	Mean	Deviation around the mean	Return	Mean	Deviation around the mean	
Sample 1	5%	3.6%	**1.4%**	1%	2%	**-1.0%**	**-0.00014**
Sample 2	-10%	3.6%	**-13.6%**	13%	2%	**11.0%**	**-0.01496**
Sample 3	12%	3.6%	**8.4%**	3%	2%	**1.0%**	**0.00084**
Sample 4	7%	3.6%	**3.4%**	-8%	2%	**-10.0%**	**-0.00340**
Sample 5	4%	3.6%	**0.4%**	1%	2%	**-1.0%**	**-0.00004**
						Summation	**-0.01770**

Covariance = -0.0177 / (5 -1) = -0.00443

Although covariance shows whether the two variables move together, it is difficult to interpret the figure. It is therefore better to convert the covariance into correlation coefficient:

$$\boldsymbol{Correlation\ coefficient_{12}} = \frac{\boldsymbol{Covariance_{12}}}{\boldsymbol{Standard\ deviation_1} \times \boldsymbol{Standard\ deviation_2}}$$

If the correlation coefficient is +1, the two variables are perfectly positively correlated (i.e. move exactly together). Conversely, if the correlation coefficient is -1, the two variables are then perfectly negatively correlated (i.e. move exactly in opposite direction).

Example:

Following the above example, calculate the correlation coefficient of the returns of Stocks A and B.

Stock B	Return	Mean	Deviation around the mean	Squared deviation
Sample 1	1%	2%	-1.0%	0.0001
Sample 2	13%	2%	11.0%	0.0121
Sample 3	3%	2%	1.0%	0.0001
Sample 4	-8%	2%	-10.0%	0.0100
Sample 5	1%	2%	-1.0%	0.0001
			Summation of squared deviation	**0.0224**

Variance $_{\text{Stock B}}$ = 0.0224 / (5 -1) = 0.0056

Standard deviation $_{\text{Stock B}}$ = $\sqrt{0.0056}$ = 7.48%

Correlation coefficient = (-0.00443) / [(8.2%)(7.48%)] = -0.72

The returns of Stocks A and B are therefore negatively correlated.

The final step is the calculation of the **portfolio variance** (or standard deviation) which represents the risk of the portfolio. The portfolio variance can be found by the following formula:

$$\boldsymbol{Portfolio\ variance = \left(w_1{}^2 SD_1{}^2 + w_2{}^2 SD_2{}^2 + 2w_1w_2 Covariance_{12}\right)}$$

The portfolio standard deviation will then be the square root of the portfolio variance.

Example:

Following the above example, calculate the portfolio (30% Stock A and 70% Stock B) standard deviation.

$$Portfolio\ standard\ deviation =$$
$$\sqrt{(0.3^2)(8.2\%^2) + (0.7^2)(7.48\%^2) + 2(0.3)(0.7)(-0.00443)} = 3.86\%$$

The above examples demonstrate the benefit of diversification. The standard deviations of the returns of Stocks A and B are 8.2% and 7.48% respectively. However, the portfolio standard deviation is 3.86% which is significantly less than the individual stock's standard deviations. Diversification works well when the returns are not positively correlated as the higher returns on one investment are offset by the low returns on another investment. In fact, provided that the returns are not perfectly correlated (i.e. correlation coefficient = +1), the portfolio standard deviation will be less than the standard deviation of the returns of individual stock. The lower the correlation coefficient, the greater the effect of the risk reduction and the portfolio standard deviation reaches the minimum when the two returns produce a correlation coefficient of -1.

This portfolio management concept was tested in the June 2010 examination in which candidates were required to explain why it is unwise to invest only in stocks from the banking and property sectors in Hong Kong. The reason is that the two sectors are closely correlated and it is hard to create a portfolio in which the returns on one sector are offset by those in the other.

Examination focus

Students are reminded they need to understand measuring risk and return as this has always been a weak area in previous examinations. In their calculations, candidates frequently make mistakes in distinguishing whether the variance or standard deviation (the square root of the variance) should be applied in the various formulae. They are therefore reminded to make good use of the information provided in the case (i.e. whether the information given is related to variance or standard deviation on the returns) before plugging figures into the formula for their calculations.